Coalitions, voting power, and political instability

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- Coalition-formation as a prisoner's dilemma: a potential theoretical explanation for political instability
- Mathematical and statistical models of voting power
- Both topics involve:
- Collaboration with Francis Tuerlinckx, Joe Bafumi, and Jonathan Katz

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Coalitions and political instability

First part of the talk

Coalitions in an election with 9 voters

- Several possibilities:
  - No coalitions
  - Single coalition of 9 voters
  - Single coalition of 3 voters
  - Three coalitions of 3 voters each
- Compute Pr (voter is decisive) for:

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Coalition-formation as a "prisoner's dilemma"

Payoffs (in terms of your own voting power) from coalition:

	Have other voters formed coalitions?	
Your option	No	Yes
Stay alone	Moderate	Very low
Join a coalition	High	Low

- Joining a coalition increases your voting power
- But if all voters form a coalition, they all have low voting power
- Suboptimal outcome if all players act rationally

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Voting power is not a zero-sum game!

- Consider 2 extremes
- Everyone votes
  - Pr (decisive vote) = Pr (election is tied)
    Assuming random voting, this probability is proportional to $1/\sqrt{n}$
- One voter is chosen at random and gets to decide the outcome
- But both systems are "fair"!

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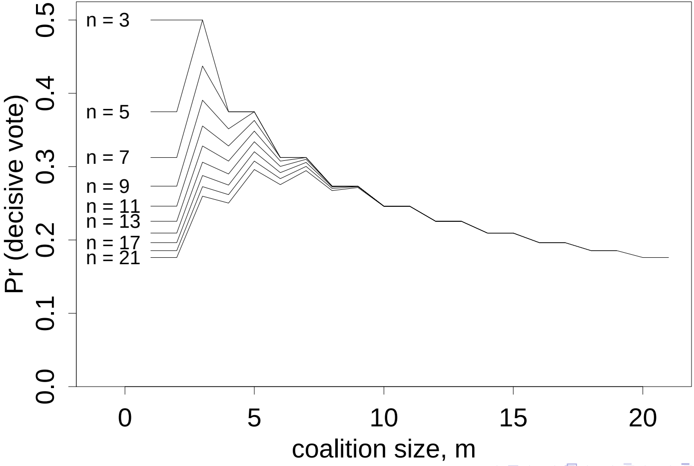
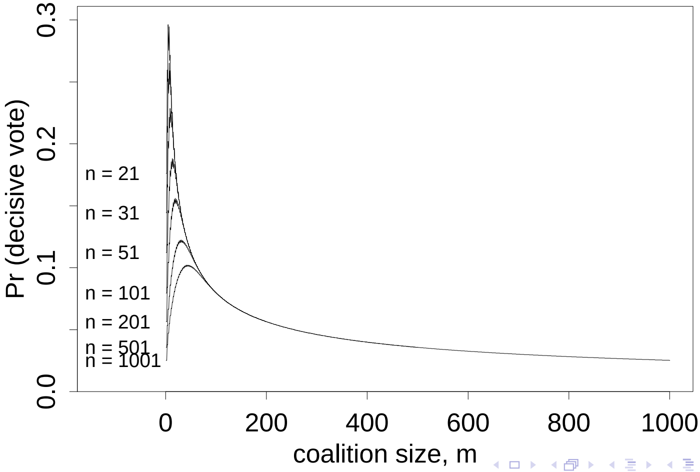
Voting power if you are in a coalition of size *m* in an electorate of size *n*



Voting power if you are in a coalition of size m in an electorate of size n (for larger populations)



Optimal coalition size m in an electorate of size n

- Using combinatorics, derive that optimal m is approx $1.4\sqrt{n}$
- Optimal coalition sizes:
  - $n = 10$ (faculty committee or student club): $m_{opt} = 4$
  - $n = 100$ (U.S. Senate): $m_{opt} = 14$
  - $n = 435$ (House of Representatives): $m_{opt} = 29$
  - $n = 5{,}000{,}000$ (Pennsylvania): $m_{opt} = 3{,}000$
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- Picture of a coalition structure as a tree
- Possible moves in tree-space:
  - Several voters forming a coalition
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A potential explanation for political instability

- Locally beneficial moves are nontransitive
- Similar to cartels in economics
- How easily can an individual voter compute Δ Pr (decisive), in order to decide whether a particular move is a good idea?
- Approximate calculations (similar to "expected utility" calculations for economic actors)
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- Local calculations of changes in voting power
- Generalizing the model:
- Fitting to real data (legislative committees, courts, political parties, . . .)

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- Local calculations of changes in voting power
- Generalizing the model:
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Mathematical and statistical models of voting power

Second part of the talk

Voting power in a 2-level electoral system

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- Electoral college:
  - n_j voters in state j
    e_j electoral votes contribute to U.S. total
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- Voting power = Pr (your vote is decisive):
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- Simple calculation based on random voting model:
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- Voters are flipping coins
- Implies that voting power $\propto e_j/\sqrt{n_j}$
- Focus on voting power as a function of n_j
- Key prediction of random voting model:
- Classical voting power calculations can also be defined combinatorially
  (Mathematically, same as the random voting model)
- But does the model describe reality?

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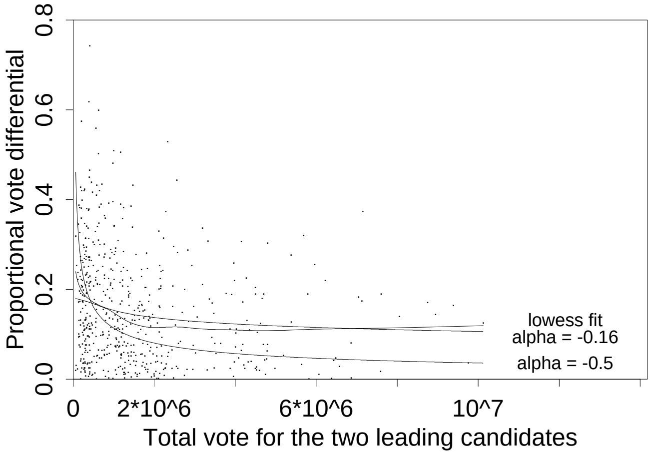
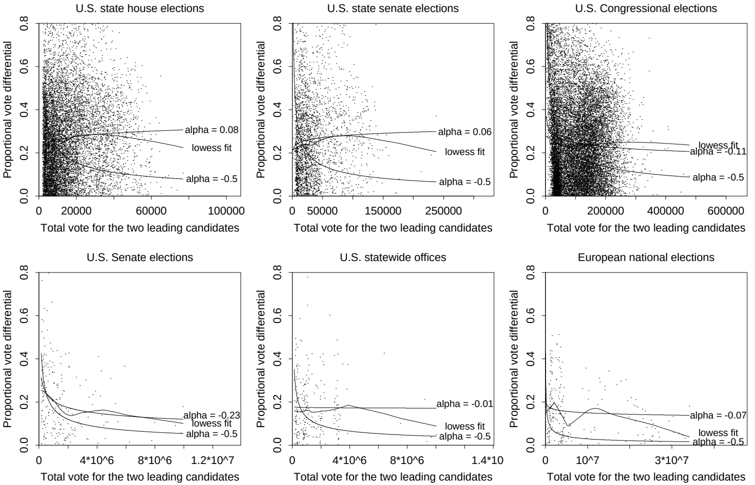
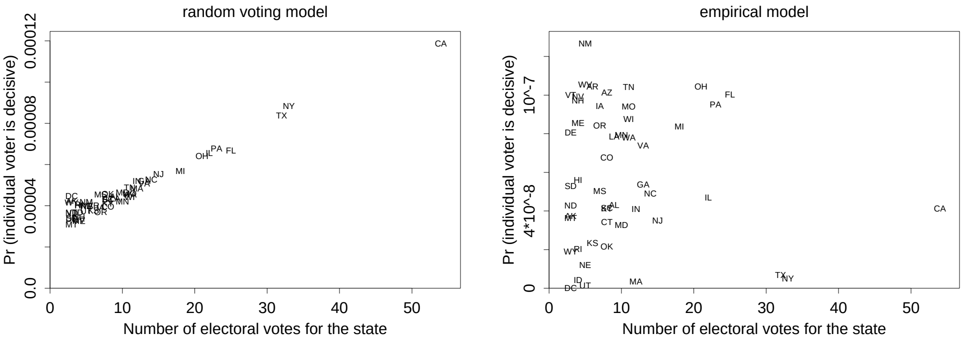
Checking the random voting model



More checks of the random voting model



Voting power by state, 2000 (math and stat models)



Voters in large states do *not* have an electoral college advantage!

More realistic models using forecasting

- A voter in Ohio or Florida has more voting power (is more likely to cast a decisive vote) than a voter in Utah or Texas
- Estimate Pr (decisive vote) in each state and each year using hierarchical time-series cross-sectional forecasting model
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- Simplest model: votes are independent with probability p instead of 1/2
  - This model is useless: it still predicts a standard deviation that is tiny when n is large
  - We must take the next step and allow votes to be correlated
- Probability models on trees (voters within neighborhoods within cities within states within regions within a country)
- Two tree models: Ising model and latent Gaussian model

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Ising model on a tree of voters

- Put a 1 or -1 at each node of the tree; correlations along branches
- Look at the implied probability distribution for $\overline{V}_n$, the average of n voters
- Standard deviation of $\overline{V}_n$ has the form $cn^{-\alpha}$
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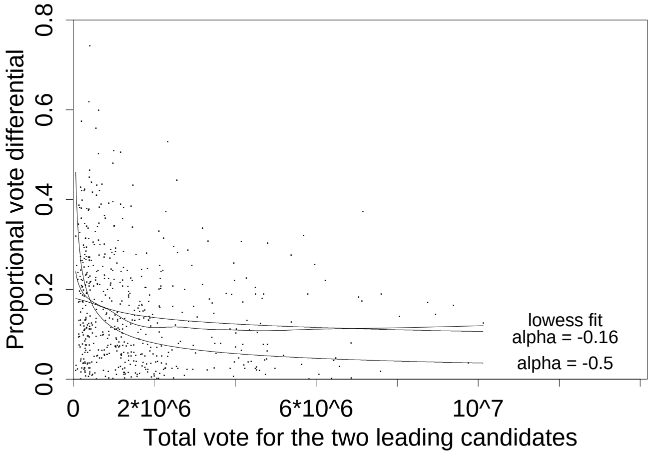

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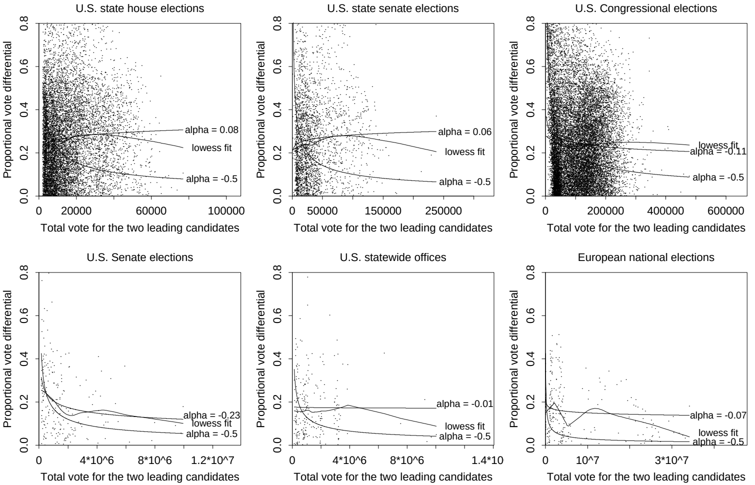

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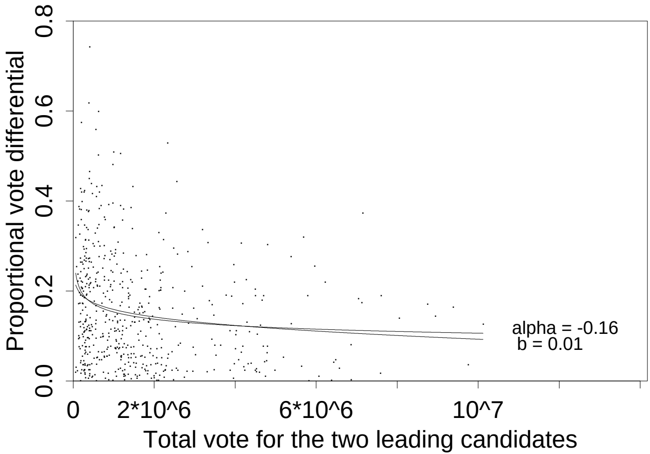
Comparing the models $cn^{-\alpha}$ and $\sqrt{a - b \log n}$



Open problems

- General statements about voting power (for example, under what conditions are proportional voting systems roughly "fair"?)
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  - Forming coalitions can help you but hurt others
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