Mathematical vs. statistical models in social science

Andrew Gelman
Department of Statistics and Department of Political Science
Columbia University

24 Oct 2005

Themes

- Mathematical models in social science are cool . . .
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz

Themes

- Mathematical models in social science are cool . . .
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz

Themes

- Mathematical models in social science are cool . . .
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz

Themes

- Mathematical models in social science are cool . . .
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz

Themes

- Mathematical models in social science are cool . . .
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz

Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting

Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting

Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting

Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting

Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting

Part 1: political representation

What does it mean to be "represented"?

- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?

What does it mean to be "represented"?

- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?

What does it mean to be "represented"?

- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?

What does it mean to be "represented"?

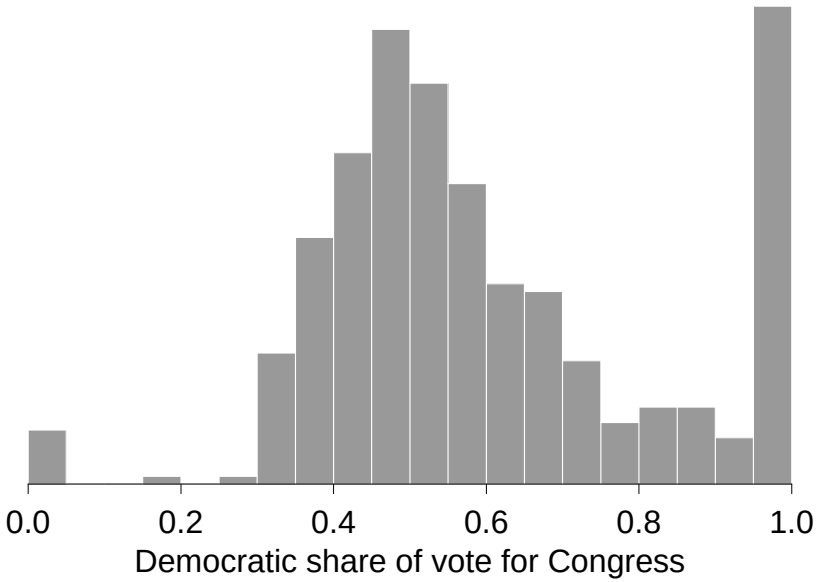
- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?

What does it mean to be "represented"?

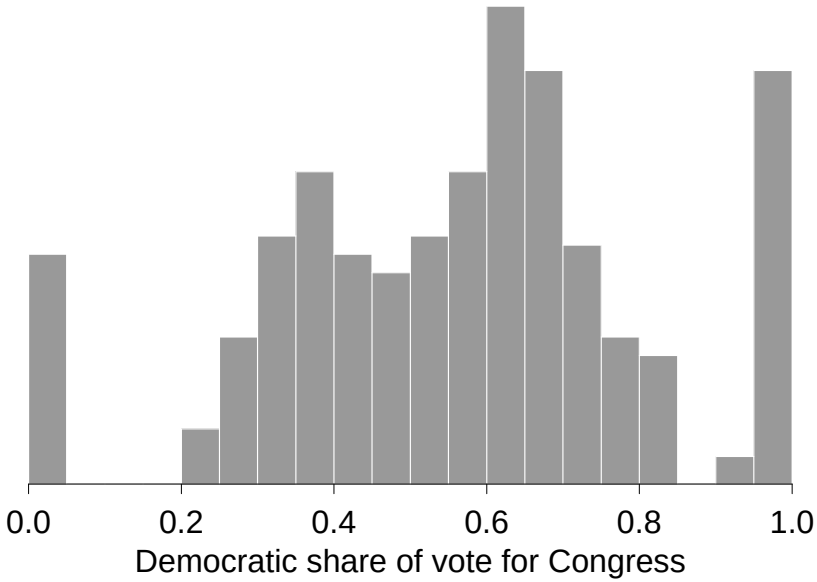
- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?

Congressional elections in 1948 and 1988

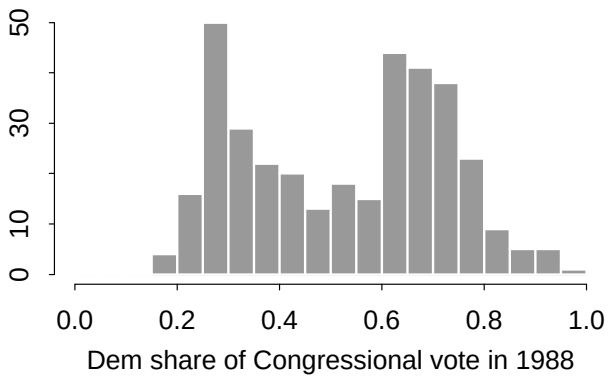
U.S. Congressional districts in 1948

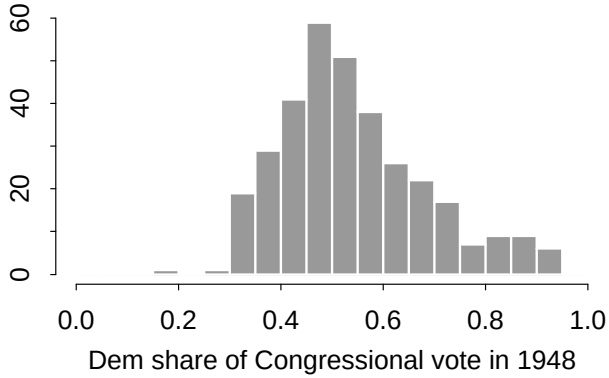


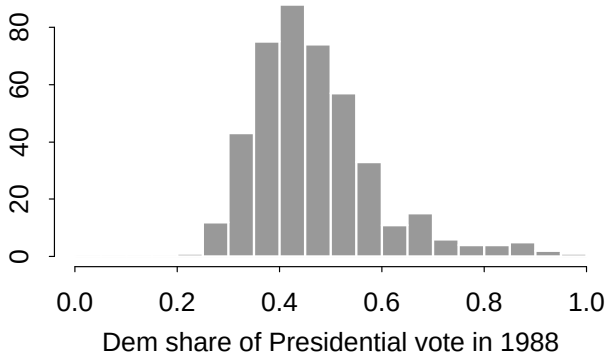
U.S. Congressional districts in 1988



Comparing to votes for President







What does it mean to be "represented"?

- Equal votes, satisfaction with outcomes, having your vote potentially matter
- Are your political views represented?
- Do your representatives look like you? Data from 1989?

	Proportion of U.S. population	Proportion of seats in House of Representatives
Catholic	0.28	0.27
Methodist	0.04	0.14
Jewish	0.02	0.07
Black	0.12	0.09
Female	0.51	0.06
Under 25	0.37	0

What does it mean to be "represented"?

- Equal votes, satisfaction with outcomes, having your vote potentially matter
- Are your political views represented?
- Do your representatives look like you? Data from 1989?

	Proportion of U.S. population	Proportion of seats in House of Representatives
Catholic	0.28	0.27
Methodist	0.04	0.14
Jewish	0.02	0.07
Black	0.12	0.09
Female	0.51	0.06
Under 25	0.37	0

What does it mean to be "represented"?

- Equal votes, satisfaction with outcomes, having your vote potentially matter
- Are your political views represented?
- Do your representatives look like you? Data from 1989?

	Proportion of U.S. population	Proportion of seats in House of Representatives
Catholic	0.28	0.27
Methodist	0.04	0.14
Jewish	0.02	0.07
Black	0.12	0.09
Female	0.51	0.06
Under 25	0.37	0

What does it mean to be "represented"?

- Equal votes, satisfaction with outcomes, having your vote potentially matter
- Are your political views represented?
- Do your representatives look like you? Data from 1989?

	Proportion of U.S. population	Proportion of seats in House of Representatives
Catholic	0.28	0.27
Methodist	0.04	0.14
Jewish	0.02	0.07
Black	0.12	0.09
Female	0.51	0.06
Under 25	0.37	0

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the "best" system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Research problem: unequal representation across the world

- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov't funding
- Look at other countries: small states/provinces are generally overrepresented
- Small states/provinces get more than their share of funds
- Larger consequences?

Part 2: trench warfare

Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man's-land
- How to understand this?
- Prisoner's dilemma

Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man's-land
- How to understand this?
- Prisoner's dilemma

Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man's-land
- How to understand this?
- Prisoner's dilemma

Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man's-land
- How to understand this?
- Prisoner's dilemma

Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man's-land
- How to understand this?
- Prisoner's dilemma

Prisoner's dilemma for trench warfare

- Payoffs in the "game" (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model

Prisoner's dilemma for trench warfare

- Payoffs in the "game" (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model

Prisoner's dilemma for trench warfare

- Payoffs in the "game" (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model

Prisoner's dilemma for trench warfare

- Payoffs in the "game" (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model

Prisoner's dilemma for trench warfare

- Payoffs in the "game" (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model

Refuting the prisoner's dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the "game" to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides

Refuting the prisoner's dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the "game" to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides

Refuting the prisoner's dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the "game" to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides

Refuting the prisoner's dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the "game" to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides

Refuting the prisoner's dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the "game" to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides

Why was the prisoner's dilemma model appealing?

- "The evolution of cooperation"
- Using game theory to solve the "tragedy of the commons"
- Axelrod's theory: politically liberal or conservative?
- "The norm of self-interest" (Miller)

Why was the prisoner's dilemma model appealing?

- "The evolution of cooperation"
- Using game theory to solve the "tragedy of the commons"
- Axelrod's theory: politically liberal or conservative?
- "The norm of self-interest" (Miller)

Why was the prisoner's dilemma model appealing?

- "The evolution of cooperation"
- Using game theory to solve the "tragedy of the commons"
- Axelrod's theory: politically liberal or conservative?
- "The norm of self-interest" (Miller)

Why was the prisoner's dilemma model appealing?

- "The evolution of cooperation"
- Using game theory to solve the "tragedy of the commons"
- Axelrod's theory: politically liberal or conservative?
- "The norm of self-interest" (Miller)

Why was the prisoner's dilemma model appealing?

- "The evolution of cooperation"
- Using game theory to solve the "tragedy of the commons"
- Axelrod's theory: politically liberal or conservative?
- "The norm of self-interest" (Miller)

Toward the future

- How to defuse future conflicts?
- Axelrod's logic: set up repeated-play structures to motivate long-term cooperation
- Alternative strategy: set up immediate gains from cooperations and watch out for outside agents who could disrupt the cooperation

Toward the future

- How to defuse future conflicts?
- Axelrod's logic: set up repeated-play structures to motivate long-term cooperation
- Alternative strategy: set up immediate gains from cooperations and watch out for outside agents who could disrupt the cooperation

Toward the future

- How to defuse future conflicts?
- Axelrod's logic: set up repeated-play structures to motivate long-term cooperation
- Alternative strategy: set up immediate gains from cooperations and watch out for outside agents who could disrupt the cooperation

Toward the future

- How to defuse future conflicts?
- Axelrod's logic: set up repeated-play structures to motivate long-term cooperation
- Alternative strategy: set up immediate gains from cooperations and watch out for outside agents who could disrupt the cooperation

Part 3: rational voting

Rational model for voting

- Utility of voting $= pB - c$:
  - $p =$ probability that a single vote will be decisive
  - $B =$ net benefit from your candidate winning
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Rational model for voting

- Utility of voting $= pB - c$:
  - p = probability that a single vote will be decisive
  - B = net benefit from your candidate winning
  - c = met cost of voting (whether or not your candidate wins)
- Paradox of voting: p is very small, so even for large values of B, there is no "instrumental" benefit to voting
- In presidential elections, p is about 1 in 10 million

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
- Poor estimation of p
- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p

- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p
- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p
  - Estimation would have to be *really* poor for p to be large enough
- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p
  - Estimation would have to be *really* poor for p to be large enough
- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p
  - Estimation would have to be *really* poor for p to be large enough
- Is voting irrational?

Possible explanations for voting

- Utility of voting $= pB - c$
- "Benefit" of voting or "civic duty"
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of p
  - Estimation would have to be *really* poor for p to be large enough
- Is voting irrational?

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - $B_{\text{self}} =$ individual benefit from candidate A winning
  - $B_{\text{soc}} =$ (your perception of) avg. benefit of others from candidate A winning
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Voting to benefit others

- Utility of voting $= pB - c$
- $B = B_{\text{self}} + \alpha N B_{\text{soc}}$
  - B_{self} = individual benefit from candidate A winning
  - B_{soc} = (your perception of) avg. benefit of others from candidate A winning
  - α (probably less than 1) discounts benefits to others
  - N = number of persons affected by the election
- It can now be rational to vote!
- Decoupling *rationality* from *selfishness*

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range ±6%
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is $10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is $10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"
  - Suppose $n/N = 1/3$ and suppose that the benefit to others (as you perceive it) is \$10 each
  - The effect of your vote on their expected gain is $\$10N/(0.12n) = \$250$

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"
  - Suppose $n/N = 1/3$ and suppose that the benefit to others (as you perceive it) is \$10 each
  - The effect of *your vote* on *their expected gain* is $\$10N/(0.12n) = \$250$
  - Voting is like making a \$250 charitable contribution

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"
  - Suppose $n/N = 1/3$ and suppose that the benefit to others (as you perceive it) is \$10 each
  - The effect of *your vote* on *their expected gain* is $\$10N/(0.12n) = \$250$
  - Voting is like making a \$250 charitable contribution

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"
  - Suppose $n/N = 1/3$ and suppose that the benefit to others (as you perceive it) is \$10 each
  - The effect of *your vote* on *their expected gain* is $\$10N/(0.12n) = \$250$
  - Voting is like making a \$250 charitable contribution

Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range $\pm 6\%$
  - Pr (your vote is decisive) $\approx 1/(0.12n)$
- Suppose the selfish benefit to you is \$10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a "social voter"
  - Suppose $n/N = 1/3$ and suppose that the benefit to others (as you perceive it) is \$10 each
  - The effect of *your vote* on *their expected gain* is $\$10N/(0.12n) = \$250$
  - Voting is like making a \$250 charitable contribution

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote

Empirical tests

- Are altruistic people more likely to vote?
- Is turnout higher in U.S. Senate elections in small states?
- Is turnout higher in NYC when there is heavy snow in Buffalo?
- Studying corporate contributions (Ansolabehere)

Empirical tests

- Are altruistic people more likely to vote?
- Is turnout higher in U.S. Senate elections in small states?
- Is turnout higher in NYC when there is heavy snow in Buffalo?
- Studying corporate contributions (Ansolabehere)

Empirical tests

- Are altruistic people more likely to vote?
- Is turnout higher in U.S. Senate elections in small states?
- Is turnout higher in NYC when there is heavy snow in Buffalo?
- Studying corporate contributions (Ansolabehere)

Empirical tests

- Are altruistic people more likely to vote?
- Is turnout higher in U.S. Senate elections in small states?
- Is turnout higher in NYC when there is heavy snow in Buffalo?
- Studying corporate contributions (Ansolabehere)

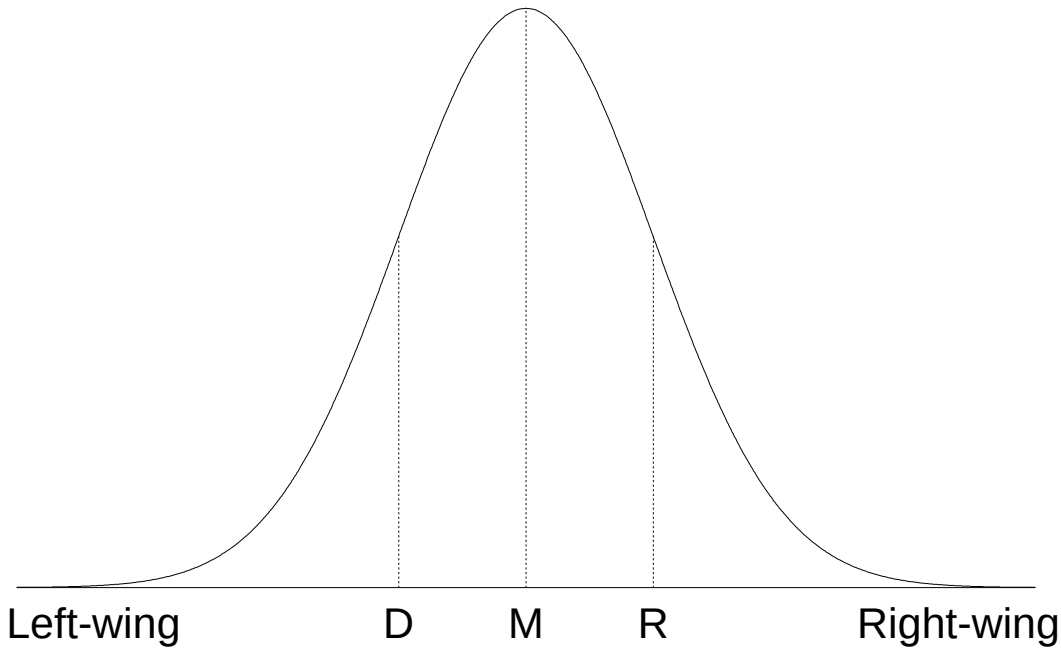
Empirical tests

- Are altruistic people more likely to vote?
- Is turnout higher in U.S. Senate elections in small states?
- Is turnout higher in NYC when there is heavy snow in Buffalo?
- Studying corporate contributions (Ansolabehere)

Part 4: candidate positioning

Candidate positioning

The "median voter theorem" (Hotelling, 1928):



Median voters and Newt Gingrich

- In the 1994 election, the Republicans gained about 50 seats in Congress
- The Democrats who lost were mostly moderate-to-conservative
- The liberal Democratic congressmembers were reelected
- Democrats should be liberal and be proud?

Median voters and Newt Gingrich

- In the 1994 election, the Republicans gained about 50 seats in Congress
- The Democrats who lost were mostly moderate-to-conservative
- The liberal Democratic congressmembers were reelected
- Democrats should be liberal and be proud?

Median voters and Newt Gingrich

- In the 1994 election, the Republicans gained about 50 seats in Congress
- The Democrats who lost were mostly moderate-to-conservative
- The liberal Democratic congressmembers were reelected
- Democrats should be liberal and be proud?

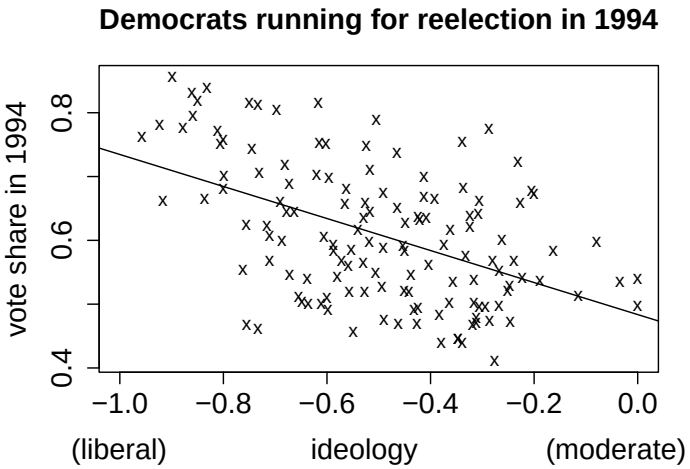
Median voters and Newt Gingrich

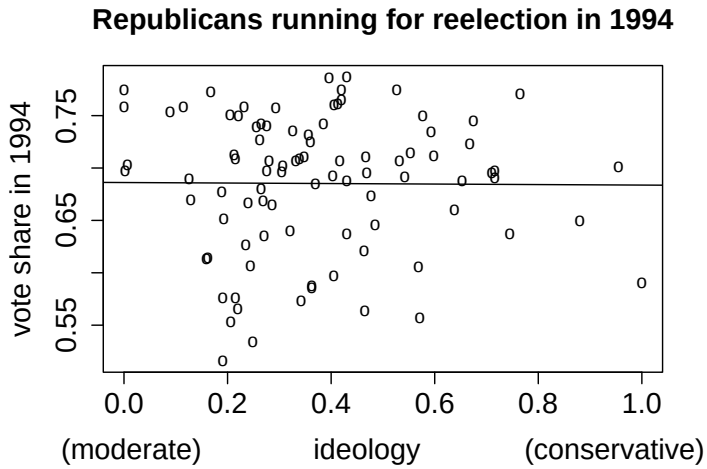
- In the 1994 election, the Republicans gained about 50 seats in Congress
- The Democrats who lost were mostly moderate-to-conservative
- The liberal Democratic congressmembers were reelected
- Democrats should be liberal and be proud?

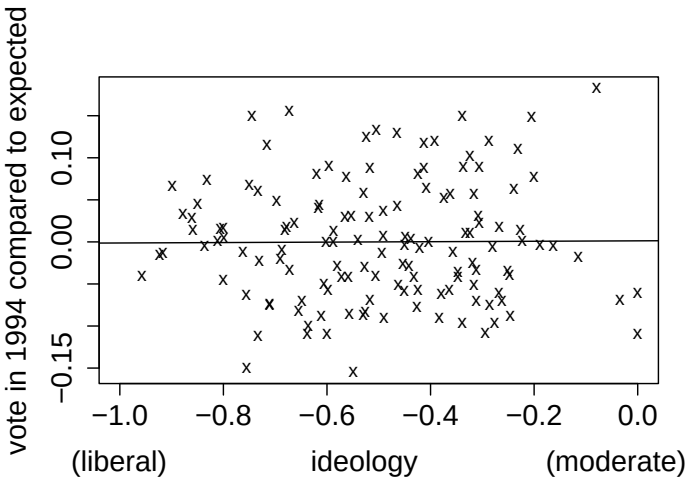
Median voters and Newt Gingrich

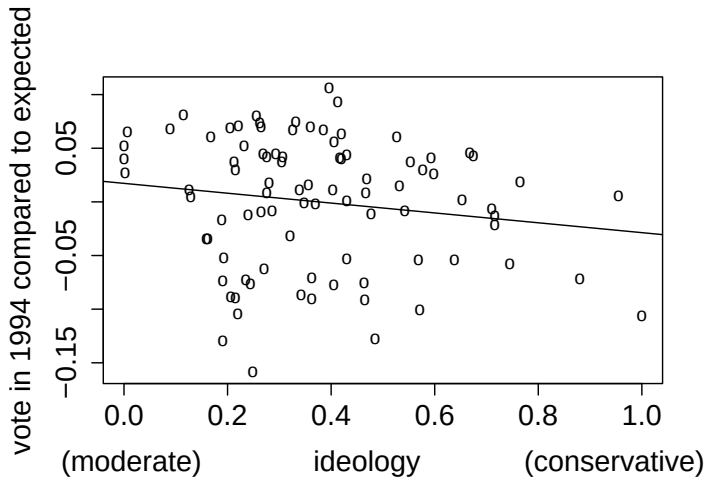
- In the 1994 election, the Republicans gained about 50 seats in Congress
- The Democrats who lost were mostly moderate-to-conservative
- The liberal Democratic congressmembers were reelected
- Democrats should be liberal and be proud?

Looking at the 1994 election more carefully

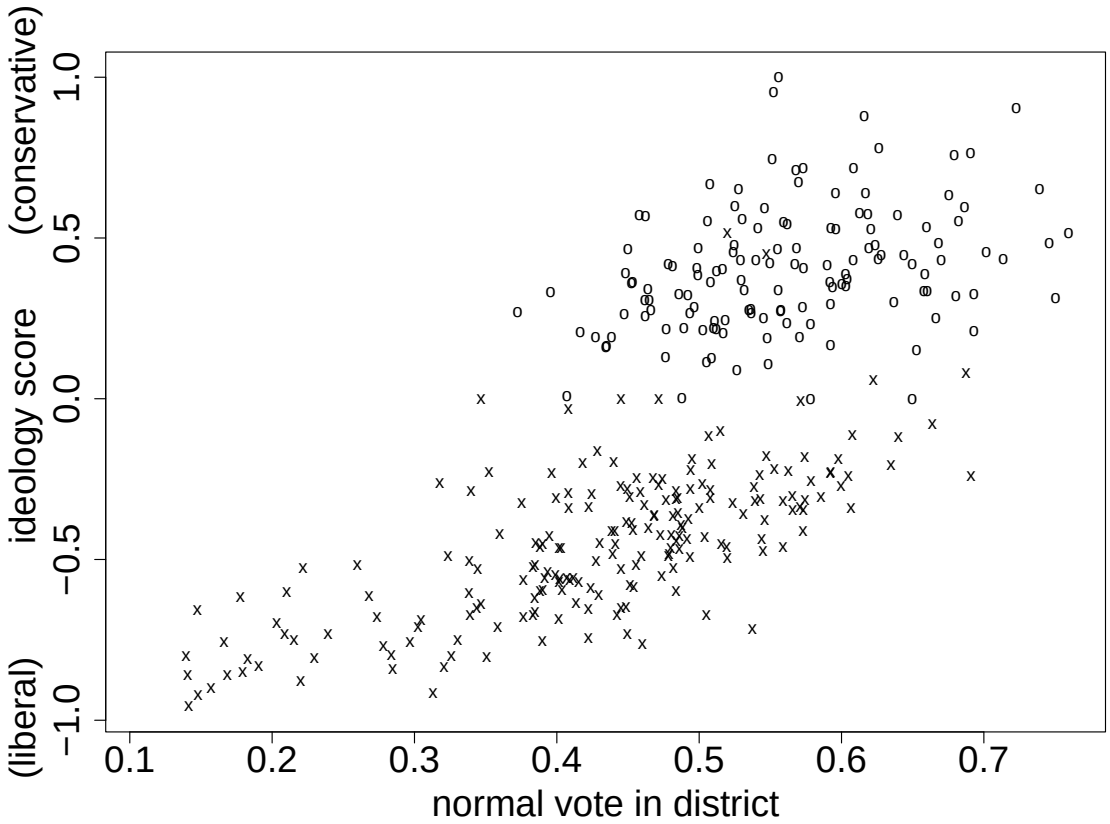








Congressmembers' ideologies and median voters



Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

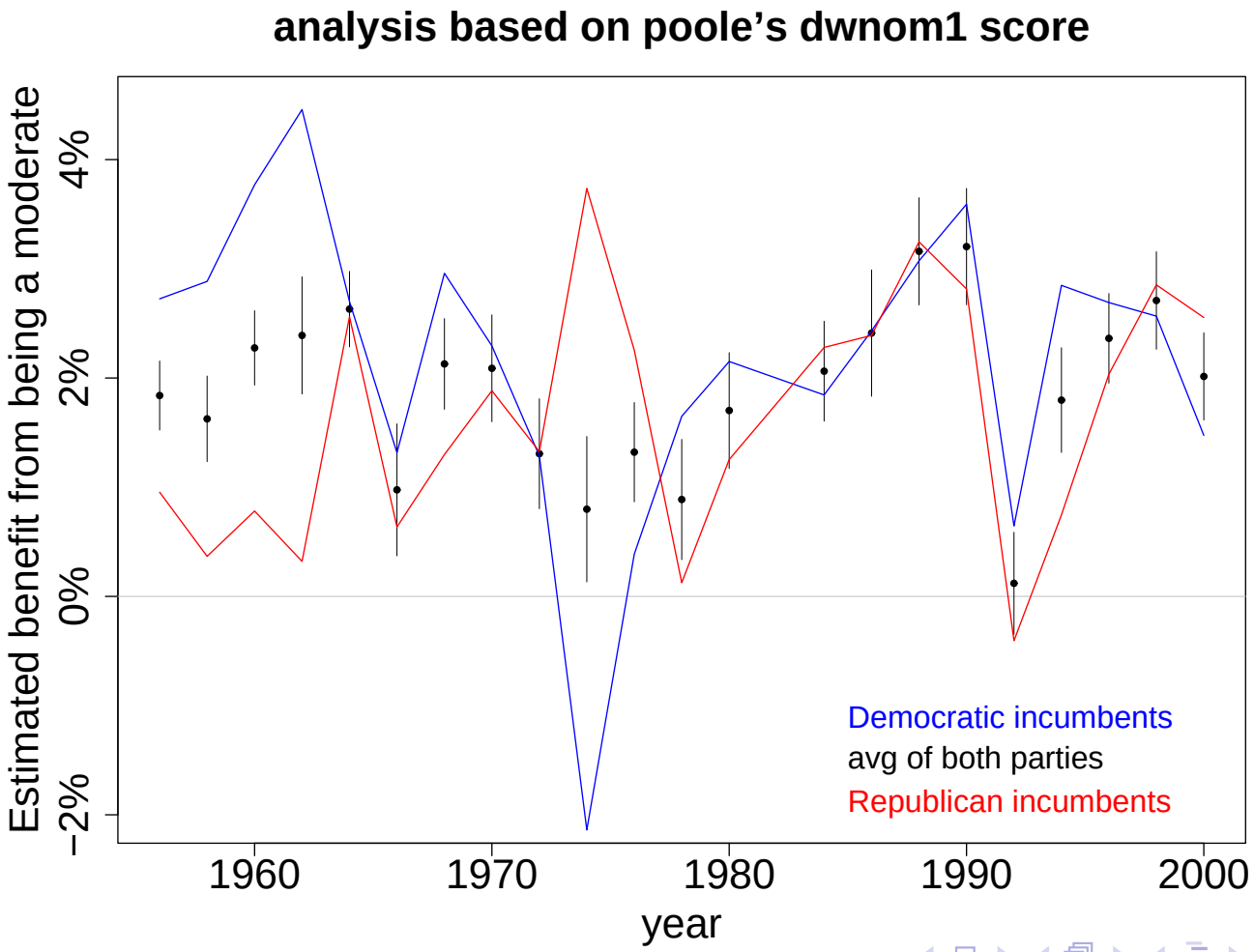
Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
- Also look at Nixon, Clinton impeachments

Estimated effects of moderation for reelection vote



Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes . . .
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes . . .
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes ...
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes ...
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes ...
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Return to the median voter theorem

- Is the median voter theorem "true"?
- No, and yes ...
- Systematic differences between Democrats and Republicans, even in comparable districts
- Moderation is worth about 2% of the vote: some motivation to be in the median, but not a lot
- Bush's gamble in 2001–2004 (and Truman's in 1945–1948): how does ideology map to policy?

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, . . .

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, ...

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, . . .

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, . . .

Summary: mathematical models in social science

- 4 examples where mathematical models gave "normative" conclusions:
  - Proportional representation is fair
  - Cooperation is a good strategy in the repeated prisoner's dilemma
  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median
- Each theory had big holes
- Each theory's predictions were essentially *qualitative*
- Statistical models take the next step
- Similar ideas hold in psychology, sociology, economics, . . .