Some computational and modeling issues for hierarchical models

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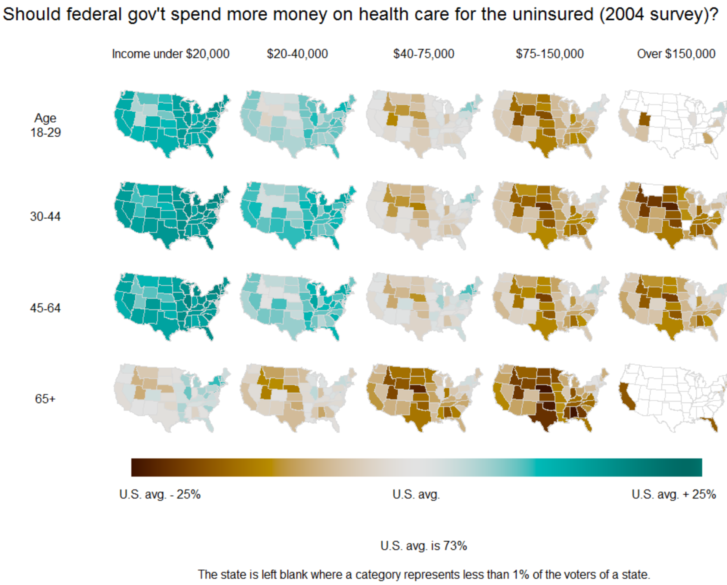
- Also can have group-level predictors and nonnested grouping factors

Application: public opinion in population subgroups



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- What's missing?
  - Something in between "automatic" and "program it yourself"

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  - Solution: allow the sophisticated user/developer to "get under the hood" and fix problems

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- And it gets worse when dimension > 2

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xi.a ~ dnorm (0, .01)
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    - Scaled-inverse-Wishart priors for covariances: $\Sigma_B = \text{Diag}(\xi) Q \text{Diag}(\xi)$

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  - No easy way to write this in Bugs or to program it oneself!
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  - Implicit graphical structure for model checking: y—θ—y^{rep}

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  - A better modeling language?

Vos pensées??

- Where to go on Bugs?
- How to work efficiently when so many research groups around the world are fitting these models?
- How to move from "The program converged!" to "The model makes sense"?