A Catch-22 in Assigning Primary Delegates

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As the 2008 election looms ever closer, states are maneuvering in various ways to try to gain increased influence. The goal for New York, as a March 31, 2007 editorial in the *New York Times* put it, should be to (i) "have a chance to play a major role in choosing their party's nominee," and (ii) "make the candidates address New York's issues." The editorial continues, "The candidates always rave about ethanol in Iowa. If they had to hunt for votes in the Bronx and on Long Island, they might talk more about mass transportation and middle-income housing."

The principal action taken by New York's legislature to accomplish these goals was to move the date of the primary election forward from March to February. In the past, much of the political dust had already settled by the time New York had its primary, and so there were few practical consequences except to second what had already been decided by the earlier states. 2008 will be different as the biggest states have moved their primaries to February, setting up a "Super Tuesday" when it is likely that the candidates for the two major parties will be determined.

But will merely moving the primary date forward be sufficient? *The Times* does not think so. The editorial argues,

> *"it will still be an easy state for campaigns to skip. For starters, there are already home-team favorites — Senator Hillary Clinton for the Democrats and former Mayor Rudolph Giuliani for Republicans.*
>
> *The more entrenched problem is New York's rules. In the Republican primary, the winner of the popular vote gets all the delegates. If Senator John McCain gets, say, 40 percent to Mr.*

Giuliani's 50 percent, he would win zero delegates. Given how expensive it is to run in New York, many candidates will probably not bother.

The Democrats' selection process is better but not much. Candidates are awarded delegates proportionately, but by Congressional districts"

If awarding delegates in an all-or-none manner, whether for the state as a whole (Republicans) or by congressional district (Democrats), is not going to aid in attaining the goals of giving New York its fair share of influence, what would be better? The *Times* suggests,

"In New York, Gov. Eliot Spitzer, the Legislature and the parties should fix the state's arcane rules. Ballot access should be simplified, and delegates should be awarded based on shares of the statewide vote."

The democratic notion of proportional representation sounds good, but will it work? To get a little ahead of ourselves, maybe not. But that sort-of conclusion awaits a more careful examination, which we explore using a small example previously described by Gelman, Katz & Tuerlinckx (2002). It is this example that provides us with the insights that form the core of our conclusions.

The example of nine voters

Suppose we are one of just nine voters and we would like to maximize the chance of our vote making a difference. What should we do? Let us begin with a definition of what it means to have our vote count. In this instance, let us say that our vote counts if we cast the deciding vote. So, if the other 8 voters are divided 8 to 0, it doesn't matter how we vote. In fact it will only matter if the other eight are divided 4 to 4. What is the probability of this happening? Let's work with the simplest statistical model and assume that all combinations of votes are equally likely, that is, each voter has a 50% chance of choosing each candidate. Then the probability of the 8 other voters dividing 4 to 4 is simply:

$(\frac{1}{2})^8$ x the number of ways that 4 votes can be taken from 8

or

(1/256) x (8x7x6x5)/(4x3x2) = 70/256 = 0.273.   (1)

Is there a way to increase this? Suppose we get together with four other voters and form a bloc. We agree that we will vote among ourselves first, and however the majority decides is the way we will cast all five votes. By forming this bloc we make the remaining four voters irrelevant. But what is the chance of our vote counting now? Simply put, our vote will count if the other 4 voters in our bloc divide 2 to 2. The probability of this happening is:

$(\frac{1}{2})^4$ x the number of ways that 2 votes can be taken from 4

or

(1/16) x (4x3/2) = 12/32 = 0.375.   (2)

This simple example has provided the insight to see why voting as a bloc increases our influence. Thinking back to the allocation of delegates we can see that if any state decides to allocate their delegates proportional to the popular vote, that state will yield an advantage of perhaps only one or two delegates to the winner over the loser. If it is an expensive state to campaign in (such as California or New York) a sensible strategy would be to skip that state and spend maximal effort in an all-or-none state where the potential rewards are far greater.

Is this all? No. Our nine-vote example has further riches that have yet to be mined. Suppose we cannot get five other voters to form a bloc, and instead all we could get was three. What are the chances of our vote counting now?

This calculation has two parts. The first part is the chance of our vote counting in our bloc. That will happen if the other two voters split 1-1, which will happen $\frac{1}{2}$ the time. The second part is the probability of the other 6 votes splitting in such a way that our 3-vote bloc is decisive. That will happen if those other 6 votes split 4 to 2, 3 to 3, or 2 to 4.

   The chances of this happening are substantial, to wit
$(\frac{1}{2})^6$ [6x5 + 5x4) = (50/64), hence the probability of our vote counting is

(1/2) x (50/64) = 50/128 = 0.391.   (3)

This strategy yields the greatest chance of our vote counting of the three cases we have examined, and would suggest that perhaps forming less-than-certain blocs is the path toward optimization. But this sort of thinking is a snare and a delusion. Let us make the assumption that the other six voters are just as clever as we are and they decide to form blocs as well. If they form a bloc of 5 or 6 our vote will not count at all, but suppose they imitate us and form themselves into two blocs of three. Now our vote will count only when those two blocs split. The probability of such an occurrence is ½, and so the probability of our vote counting is the product of it counting within the bloc (½) and the other two blocs splitting (½) yielding a probability of 0.25.

Compare this figure with the 0.273 given in expression (1) for the probability of our vote counting in an unaligned situation. As discussed by Gelman (2003), this is directly analogous to the so-called Prisoner's Dilemma where if we don't form blocs and some others do, the power of our vote is diminished. But if we all form blocs everyone's influence is less. This is the Catch-22 we referred to in the title.

Implications for New York—and other states

Our argument suggests that, on average, voters might gain if all states were to adopt rules that gave proportional representation based on the primary vote—but if even one state demurred and opted to form a bloc by using an all-or-none allocation scheme, every state but that one would lose. This in turn would motivate each of the other states to switch to an all-or-nothing allocation to increase their own probabilities of influencing the outcome. (Blocs can also be formed by groups of states, making our example more complex by adding nested coalitions; in a sense, this is what occurs when a group of states band together to create a "Super Tuesday" of primaries on a single day, with the goal that the winner of this group of primaries will have a better chance of gaining a decisive number of delegates.)

However, these calculations depend on the assumption that voters are closely divided as to which candidate to support. To study the implications of allocation rules for actual elections, we need to go the next step and actually estimate the probability distributions using historical votes. A key factor is the closeness and anticipated closeness of the elections within

each state (see Gelman, 2003, and Gelman, Katz, and Bafumi, 2004). In particular, if the Democrats (say) think that Hillary Clinton is almost certain to win New York and all its delegates, then they might want to consider the *Times* suggestion and switch to a proportional system where all votes will be counted—although this would have the side effect of decreasing the delegate total for hometown favorite Hillary. But if there is a good chance that the primary election will be close, New Yorkers would actually have more influence with a winner-take-all system.

If New York Republicans (or Democrats) prefer a proportional system for allocating delegates, the best way to accomplish it might be to work in concert with other states so as to avoid the Prisoner's Dilemma problem that allows any state to gain by switching to a winner-take-all system. They could take a cue from the state of Maryland, which, as reported by Brian Witte in the Associated Press on April 10, 2007,

> *"officially became the first state on Tuesday to approve a plan to give its electoral votes for president to the winner of the national popular vote instead of the candidate chosen by state voters. . . . The measure would award Maryland's 10 electoral votes to the national popular vote winner. The plan would only take effect if states representing a majority of the nation's 538 electoral votes decided to make the same change."*

To beat a bloc, form a bloc of non-blockers.

Acknowledgment

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