

HW problems due April 5, 2006.

1. Let $z \sim \mathcal{N}(\mu, C)$, with

$$\mu = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

and

$$C = \begin{pmatrix} 6 & -2 & 1 \\ -2 & 3 & -1 \\ 1 & -1 & 1 \end{pmatrix}.$$

Write $p(z)$ as a product of three independent Gaussian probability densities which are of “rank 1” (that is, the densities only vary along one direction).

2. Use Cochran’s theorem to prove that the sum of the squared residuals $e^t e$ in the simple linear regression model with normal errors is distributed as a chi-squared random variable with $N - 2$ degrees of freedom (you’ll have to prove something about the rank of the quadratic form $e^t e$). Name an important quantity that $e^t e$ is independent of.